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Quantum Gauge Symmetry from Classical Gauge Non-invariant Action

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Abstract

We have recently shown that the modified quantization scheme of Zwanziger, Parrinello and Jona-Lasinio is in fact identical at least in the perturbative accuracy to the conventional Faddeev-Popov formula, if one takes into account the variation of the gauge field along the entire gauge orbit. This in particular suggests that the classical massive gauge theory, for example, and the gauge invariant theory, whose gauge symmetry is broken by a gauge fixing term, have no intrinsic differences in a suitably quantized theory. Classical gauge symmetry is sufficient to ensure quantum gauge symmetry (BRST symmetry), but it is not necessary in general. It is thus suggested to extend the notion of quantum gauge symmetry not only to classical gauge theory but also to any theory whose gauge symmetry is broken by some extra terms in the classical action. As for massive gauge particles, only the Higgs mechanics, where the mass term is gauge invariant, has an intrinsic meaning. We comment on a possible connection of the present observation to the past arguments against the dynamical generation of massless gauge fields.

1 Introduction

We have recently shown[1] that the modified quantization scheme[2][3]

$$\int \mathcal{D}A_\mu \{\exp[-S_{YM}(A_\mu) - \int f(A_\mu)dx]/\int \mathcal{D}g \exp[-\int f(A_\mu^g)dx]\} \tag{1.1}$$

with, for example,

$$f(A_\mu) = (1/2)(A_\mu)^2 \tag{1.2}$$

is identical at least in the perturbative accuracy to the conventional local Faddeev-Popov formula[4]

$$\begin{aligned}&\int \mathcal{D}A_\mu \{\delta(D^\mu \frac{\delta f(A_\nu)}{\delta A_\mu})/\int \mathcal{D}g\delta(D^\mu \frac{\delta f(A_\nu^g)}{\delta A_\mu^g})\}\exp[-S_{YM}(A_\mu)]\\ &= \int \mathcal{D}A_\mu \delta(D^\mu \frac{\delta f(A_\nu)}{\delta A_\mu})\det\{\delta[D^\mu \frac{\delta f(A_\nu^g)}{\delta A_\mu^g}]/\delta g\}\exp[-S_{YM}(A_\mu)]\end{aligned} \tag{1.3}$$

if one takes into account the variation of the gauge field along the entire gauge orbit parametrized by the gauge parameter g. Here the operator $D^{\mu}\frac{\delta f(A_{\nu})}{\delta A_{\mu}}(x)$ is defined by an infinitesimal gauge transformation as

$$\int dx f(A_{\nu}+D_{\nu}\omega)=\int dx f(A_{\nu})-\int dx\omega(x)D^{\mu}\frac{\delta f(A_{\nu})}{\delta A_{\mu}}(x). \tag{1.4}$$

The above equivalence was discussed in[1] in connection with the analysis of the so-called Gribov problem[5], and the above formula is valid if the Gribov-type complications are ignored.

We here discuss the possible implications of the above equivalence in a more general context of quantum gauge symmetry ,namely, BRST symmetry[6], which controls the analyses of renormalization, if the action contains no terms whose mass dimension is larger than 4, and unitarity. We argue that from a view point of quantum gauge symmetry there is no intrinsic difference between the classical theory with some extra gauge symmetry breaking terms such as a mass term and the gauge theory whose gauge symmetry is broken by a gauge fixing term. In particular, the classical massive Yang-Mills theory, for example, has no intrinsic differences from pure Yang-Mills theory if the theory is quantized properly.

2 Abelian example

We first briefly illustrate the proof[1] of the above equivalence of (1.1) and (1.3) by using an example of Abelian gauge theory,

$$S_0=-\frac{1}{4}\int dx(\partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu})^2 \tag{2.1}$$

for which we can work out everything explicitly. In this note we exclusively work on Euclidean theory with metric convention $g_{\mu\nu}=(1,1,1,1)$. Note that there is no Gribov complications in the Abelian theory at least in a continuum formulation. As a simple and useful example, we choose the gauge fixing function[2][3]

$$f(A)\equiv\frac{1}{2}A_{\mu}A_{\mu} \tag{2.2}$$

and

$$D_{\mu}(\frac{\delta f}{\delta A_{\mu}})=\partial_{\mu}A_{\mu}. \tag{2.3}$$

Our claim above suggests the relation

$$\begin{aligned}Z &= \int\mathcal{D}A^{\omega}_{\mu}\{e^{-S_0(A^{\omega}_{\mu})-\int dx\frac{1}{2}(A^{\omega}_{\mu})^2}/\int\mathcal{D}he^{-\int dx\frac{1}{2}(A^{h\omega}_{\mu})^2}\}\\ &= \int\mathcal{D}A^{\omega}_{\mu}\mathcal{D}B\mathcal{D}\bar{c}\mathcal{D}ce^{-S_0(A^{\omega}_{\mu})+\int[-iB\partial_{\mu}A^{\omega}_{\mu}+\bar{c}(-\partial_{\mu}\partial_{\mu})c]dx}\end{aligned} \tag{2.4}$$

where the variable A^ω_μ stands for the field variable obtained from A_μ by a gauge transformation parametrized by the gauge orbit parameter ω. To establish this result, we first evaluate

$$
\begin{aligned}
&\int \mathcal{D}h e^{-\int dx \frac{1}{2}(A^{h\omega}_\mu)^2}\\
&= \int \mathcal{D}h e^{-\int dx \frac{1}{2}(A^\omega_\mu + \partial_\mu h)^2}\\
&= \int \mathcal{D}h e^{-\int dx \frac{1}{2}[(A^\omega_\mu)^2 - 2(\partial_\mu A^\omega_\mu)h + h(-\partial_\mu\partial_\mu)h]}\\
&= \int \mathcal{D}B \frac{1}{det\sqrt{-\partial_\mu\partial_\mu}} e^{-\int dx \frac{1}{2}[(A^\omega_\mu)^2 - 2(\partial_\mu A^\omega_\mu)\frac{1}{\sqrt{-\partial_\mu\partial_\mu}}B + B^2]}\\
&= \frac{1}{det\sqrt{-\partial_\mu\partial_\mu}} e^{-\int dx \frac{1}{2}(A^\omega_\mu)^2 + \frac{1}{2}\int \partial_\mu A^\omega_\mu \frac{1}{-\partial_\mu\partial_\mu}\partial_\nu A^\omega_\nu dx}
\end{aligned}
\tag{2.5}
$$

where we defined $\sqrt{-\partial_\mu\partial_\mu}h = B$. Thus

$$
\begin{aligned}
Z &= \int \mathcal{D}A^\omega_\mu \{det\sqrt{-\partial_\mu\partial_\mu}\} e^{-S_0(A^\omega_\mu) - \frac{1}{2}\int \partial_\mu A^\omega_\mu \frac{1}{-\partial_\mu\partial_\mu}\partial_\nu A^\omega_\nu dx}\\
&= \int \mathcal{D}A^\omega_\mu \mathcal{D}B\mathcal{D}\bar{c}\mathcal{D}c e^{-S_0(A^\omega_\mu) - \frac{1}{2}\int B^2 dx + \int[-iB\frac{1}{\sqrt{-\partial_\mu\partial_\mu}}\partial_\mu A^\omega_\mu + \bar{c}\sqrt{-\partial_\mu\partial_\mu}c]dx}
\end{aligned}
\tag{2.6}
$$

which is invariant under the BRST transformation

$$
\begin{aligned}
&\delta A^\omega_\mu = i\lambda\partial_\mu c\\
&\delta c = 0\\
&\delta\bar{c} = \lambda B\\
&\delta B = 0
\end{aligned}
\tag{2.7}
$$

with a Grassmann parameter λ. Note the appearance of the imaginary factor i in the term $iB\frac{1}{\sqrt{-\partial_\mu\partial_\mu}}\partial_\mu A^\omega_\mu$ in (2.6).

We next rewrite the expression (2.6) as

$$
\begin{aligned}
&\int \mathcal{D}A^\omega_\mu \mathcal{D}B\mathcal{D}\Lambda\mathcal{D}\bar{c}\mathcal{D}c\,\delta(\frac{1}{\sqrt{-\partial_\mu\partial_\mu}}\partial_\mu A^\omega_\mu - \Lambda) e^{-S_0(A^\omega_\mu) - \frac{1}{2}\int (B^2 + 2i\Lambda B)dx + \int \bar{c}\sqrt{-\partial_\mu\partial_\mu}c dx}\\
&= \int \mathcal{D}A^\omega_\mu \mathcal{D}\Lambda\mathcal{D}\bar{c}\mathcal{D}c\,\delta(\frac{1}{\sqrt{-\partial_\mu\partial_\mu}}\partial_\mu A^\omega_\mu - \Lambda) e^{-S_0(A^\omega_\mu) - \frac{1}{2}\int \Lambda^2 dx + \int \bar{c}\sqrt{-\partial_\mu\partial_\mu}c dx}.
\end{aligned}
\tag{2.8}
$$

We note that we can compensate any variation of $\delta\Lambda$ by a suitable change of gauge parameter $\delta\omega$ inside the δ-function as

$$
\frac{1}{\sqrt{-\partial_\mu\partial_\mu}}\partial_\mu\partial_\mu\delta\omega = \delta\Lambda.
\tag{2.9}
$$

By a repeated application of infinitesimal gauge transformations combined with the invariance of the path integral measure under these gauge transformations, we can re-write the formula (2.8) as

$$\begin{aligned}
&\int \mathcal{D}A_\mu^\omega \mathcal{D}\Lambda \mathcal{D}\bar{c}\mathcal{D}c\delta(\frac{1}{\sqrt{-\partial_\mu\partial_\mu}}\partial_\mu A_\mu^\omega)e^{-S_0(A_\mu^\omega)-\frac{1}{2}\int\Lambda^2 dx+\int\bar{c}\sqrt{-\partial_\mu\partial_\mu}cdx} \\
&= \int \mathcal{D}A_\mu^\omega \mathcal{D}\bar{c}\mathcal{D}c\delta(\frac{1}{\sqrt{-\partial_\mu\partial_\mu}}\partial_\mu A_\mu^\omega)e^{-S_0(A_\mu^\omega)+\int\bar{c}\sqrt{-\partial_\mu\partial_\mu}cdx} \\
&= \int \mathcal{D}A_\mu^\omega \mathcal{D}B\mathcal{D}\bar{c}\mathcal{D}ce^{-S_0(A_\mu^\omega)+\int[-iB\frac{1}{\sqrt{-\partial_\mu\partial_\mu}}\partial_\mu A_\mu^\omega+\bar{c}\sqrt{-\partial_\mu\partial_\mu}c]dx} \\
&= \int \mathcal{D}A_\mu^\omega \mathcal{D}B\mathcal{D}\bar{c}\mathcal{D}ce^{-S_0(A_\mu^\omega)+\int[-iB\partial_\mu A_\mu^\omega+\bar{c}(-\partial_\mu\partial_\mu)c]dx}. \qquad (2.10)
\end{aligned}$$

In the last stage of this equation, we re-defined the *auxiliary* variables B and $\bar{c}$ as

$$\begin{aligned}
&B \to B\sqrt{-\partial_\mu\partial_\mu} \\
&\bar{c} \to \bar{c}\sqrt{-\partial_\mu\partial_\mu} \qquad (2.11)
\end{aligned}$$

which is consistent with BRST symmetry and leaves the path integral measure invariant. We have thus established the desired result (2.4).

It is shown that this procedure works for the non-Abelian case also[1], if the (ill-understood) Gribov-type complications can be ignored such as in perturbative calculations.

3 No massive gauge fields?

In the classical level, we traditionally consider

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2}m^2 A_\mu A^\mu \qquad (3.1)$$

as a Lagrangian for a massive vector theory, and

$$\mathcal{L}_{eff} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2}(\partial_\mu A^\mu)^2 \qquad (3.2)$$

as an effective Lagrangian for Maxwell theory with a Feynman-type gauge fixing term added. The physical meanings of these two Lagrangians are thus completely different.

However, the analysis in Section 2 shows that the Lagrangian (3.1) could in fact be regarded as a gauge fixed Lagrangian of *massless* Maxwell field in quantized theory. To be explicit, by using (2.4), the Lagrangian (3.1) may be regarded as an effective Lagrangian in

$$\begin{aligned}
Z &= \int \mathcal{D}A_\mu^\omega\{e^{\int dx[-\frac{1}{4}(\partial_\mu A_\nu-\partial_\nu A_\mu)^2-\frac{1}{2}m^2A_\mu^\omega A^{\omega\mu}]}/\int\mathcal{D}he^{-\int dx\frac{m^2}{2}(A_\mu^{h\omega})^2}\} \\
&= \int \mathcal{D}A_\mu^\omega \mathcal{D}B\mathcal{D}\bar{c}\mathcal{D}ce^{\int dx[-\frac{1}{4}(\partial_\mu A_\nu-\partial_\nu A_\mu)^2-iB\partial_\mu A_\mu^\omega+\bar{c}(-\partial_\mu\partial_\mu)c]dx}. \qquad (3.3)
\end{aligned}$$

where we absorbed the factor m^2 into the definition of B and $\bar{c}$. On the other hand, (3.2) is obtained from

$$\begin{aligned}
&\int \mathcal{D}A^\omega_\mu \mathcal{D}B\mathcal{D}\bar{c}\mathcal{D}c e^{\int dx[-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - iB\partial_\mu A^\omega_\mu + \bar{c}(-\partial_\mu\partial_\mu)c]dx} \\
&= \int \mathcal{D}A^\omega_\mu \mathcal{D}B\mathcal{D}\bar{c}\mathcal{D}c e^{\int dx[-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2}\xi B^2 - iB\partial_\mu A^\omega_\mu + \bar{c}(-\partial_\mu\partial_\mu)c]dx} \\
&= \int \mathcal{D}A^\omega_\mu \mathcal{D}\bar{c}\mathcal{D}c e^{\int dx[-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2\xi}(\partial_\mu A^\omega_\mu)^2 + \bar{c}(-\partial_\mu\partial_\mu)c]dx} \qquad (3.4)
\end{aligned}$$

and by setting the gauge parameter $\xi = 1$. The first equality of (3.4), namely, the ξ independence of the partition function is established by the BRST identity as follows: When one defines

$$Z(\xi) = \int \mathcal{D}A^\omega_\mu \mathcal{D}B\mathcal{D}\bar{c}\mathcal{D}c e^{-S_0(A^\omega) - \frac{\xi}{2}\int B^2 dx + \int[-iB\partial_\mu A^\omega_\mu + \bar{c}(-\partial_\mu\partial_\mu)c]dx} \qquad (3.5)$$

with $S_0(A^\omega)$ defined in (2.1), one can show that

$$\begin{aligned}
&Z(\xi - \delta\xi) \\
&= \int \mathcal{D}A^\omega_\mu \mathcal{D}B\mathcal{D}\bar{c}\mathcal{D}c e^{-S_0(A^\omega_\mu) - \frac{\xi-\delta\xi}{2}\int B^2 dx + \int[-iB\partial_\mu A^\omega_\mu + \bar{c}(-\partial_\mu\partial_\mu)c]dx} \\
&= Z(\xi) \qquad (3.6) \\
+ &\int \mathcal{D}A^\omega_\mu \mathcal{D}B\mathcal{D}\bar{c}\mathcal{D}c(\frac{\delta\xi}{2}\int B^2 dx) e^{-S_0(A^\omega_\mu) - \frac{\xi}{2}\int B^2 dx + \int[-iB\partial_\mu A^\omega_\mu + \bar{c}(-\partial_\mu\partial_\mu)c]dx}.
\end{aligned}$$

On the other hand,the BRST invariance of the path integral measure and the effective action in the exponential factor gives rise to

$$\begin{aligned}
&\int \mathcal{D}A^\omega_\mu \mathcal{D}B\mathcal{D}\bar{c}\mathcal{D}c[\int(\bar{c}B)dx] e^{-S_0(A^\omega_\mu) - \frac{\xi}{2}\int B^2 dx + \int[-iB\partial_\mu A^\omega_\mu + \bar{c}(-\partial_\mu\partial_\mu)c]dx} \\
&= \int \mathcal{D}A'^\omega_\mu \mathcal{D}B'\mathcal{D}\bar{c}'\mathcal{D}c'[\int(\bar{c}'B')dx] \\
&\times e^{-S_0(A'^\omega_\mu) - \frac{\xi}{2}\int B'^2 dx + \int[-iB'\partial_\mu A'^\omega_\mu + \bar{c}'(-\partial_\mu\partial_\mu)c']dx} \\
&= \int \mathcal{D}A^\omega_\mu \mathcal{D}B\mathcal{D}\bar{c}\mathcal{D}c[\int(\bar{c}B + \delta_{BRST}(\bar{c}B))dx] \\
&\times e^{-S_0(A^\omega_\mu) - \frac{\xi}{2}\int B^2 dx + \int[-iB\partial_\mu A^\omega_\mu + \bar{c}(-\partial_\mu\partial_\mu)c]dx} \qquad (3.7)
\end{aligned}$$

where the BRST transformed variables are defined by $A'^\omega_\mu = A^\omega_\mu + i\lambda\partial_\mu c$, $B' = B, \bar{c}' = \bar{c} + \lambda B, c' = c$. The first equality of the above relation means that the path integral itself is independent of the naming of integration variables, and the second equality follows from the BRST invariance of the path integral measure and the action. Namely, the BRST exact quantity vanishes as

$$\begin{aligned}
&\int \mathcal{D}A^\omega_\mu \mathcal{D}B\mathcal{D}\bar{c}\mathcal{D}c(\int \delta_{BRST}(\bar{c}B)dx) e^{-S_0(A^\omega_\mu) - \frac{\xi}{2}\int B^2 dx + \int[-iB\partial_\mu A^\omega_\mu + \bar{c}(-\partial_\mu\partial_\mu)c]dx} \\
&= \int \mathcal{D}A^\omega_\mu \mathcal{D}B\mathcal{D}\bar{c}\mathcal{D}c(\lambda\int B^2 dx) e^{-S_0(A^\omega) - \frac{\xi}{2}\int B^2 dx + \int[-iB\partial_\mu A^\omega_\mu + \bar{c}(-\partial_\mu\partial_\mu)c]dx} \\
&= 0. \qquad (3.8)
\end{aligned}$$

Thus from (3.6) we have the relation

$$Z(\xi - \delta\xi) = Z(\xi). \tag{3.9}$$

Namely, $Z(\xi)$ is independent of ξ, and $Z(1) = Z(0)$. This justifies the equality used in (3.4).

One can in fact analyze (3.2) directly by defining

$$f(A_\mu) \equiv \frac{1}{2}(\partial_\mu A^\mu)^2 \tag{3.10}$$

in the modified quantization scheme (1.1). The equality of (1.1) and (1.3) then gives

$$\begin{aligned}
&\int \mathcal{D}A_\mu \delta(D^\mu \frac{\delta f(A_\nu)}{\delta A_\mu}) \det\{\delta[D^\mu \frac{\delta f(A_\nu^g)}{\delta A_\mu^g}]/\delta g\} \exp[-S_0(A_\mu)] \\
&= \int \mathcal{D}A_\mu \delta(\partial_\nu\partial^\nu(\partial^\mu A_\mu)) \det[\partial_\nu\partial^\nu\partial_\mu\partial^\mu] \exp[-S_0(A_\mu)] \qquad (3.11)\\
&= \int \mathcal{D}A_\mu \mathcal{D}B\mathcal{D}\bar{c}\mathcal{D}c \exp\{-S_0(A_\mu) + \int dx[-iB\partial_\nu\partial^\nu(\partial^\mu A_\mu) - \bar{c}(\partial_\nu\partial^\nu\partial_\mu\partial^\mu)c]\}
\end{aligned}$$

After the re-definition of *auxiliary* variables,

$$B\partial_\nu\partial^\nu \rightarrow B, \qquad \bar{c}\partial_\nu\partial^\nu \rightarrow \bar{c} \tag{3.12}$$

which preserves BRST symmetry, (3.11) becomes

$$\int \mathcal{D}A_\mu \mathcal{D}B\mathcal{D}\bar{c}\mathcal{D}c \exp\{-S_0(A_\mu) + \int dx[-iB(\partial^\mu A_\mu) + \bar{c}(-\partial_\mu\partial^\mu)c]\} \tag{3.13}$$

which agrees with (2.10) and (3.3).

We can thus assign an identical physical meaning to two Lagrangians (3.1) and (3.2) in quantized theory.

Similarly, the two classical Lagrangians related to Yang-Mills fields

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c)^2 - \frac{m^2}{2} A_\mu^a A^{a\mu} \tag{3.14}$$

and

$$\mathcal{L}_{eff} = -\frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c)^2 - \frac{1}{2}(\partial_\mu A^{a\mu})^2 \tag{3.15}$$

could be assigned an identical physical meaning as an effective gauge fixed Lagrangian associated with the quantum theory defined by[1]

$$\int \mathcal{D}A_\mu^a \mathcal{D}B^a \mathcal{D}\bar{c}^a \mathcal{D}c^a \exp\{-S_{YM}(A_\mu^a) + \int dx[-iB^a(\partial^\mu A_\mu^a) + \bar{c}^a(-\partial_\mu(D^\mu c)^a]\} \tag{3.16}$$

which is invariant under the quantum gauge symmetry (BRST transformation) with a Grassmann parameter λ

$$\begin{aligned}
\delta A_\mu^a &= i\lambda(D_\mu c)^a \\
\delta c^a &= -\frac{i\lambda}{2} f^{abc} c^b c^c \\
\delta \bar{c}^a &= \lambda B^a \\
\delta B^a &= 0.
\end{aligned} \tag{3.17}$$

In this analysis, we ignore the (ill-understood) Gribov-type complications. This connection with the possible Gribov-type complications becomes transparent if one considers

$$f(A^{a\mu}) = \frac{1}{2}(\partial_\mu A^{a\mu})^2 \tag{3.18}$$

in (3.15). We then obtain in the modified scheme (1.1) and its equivalent formula (1.3) (by suppressing the Yang-Mills indices)

$$D^\mu \frac{\delta f(A_\nu)}{\delta A_\mu} = D_\mu \partial^\mu (\partial_\nu A^\nu) \tag{3.19}$$

and the associated determinant factor contains the operator

$$D_\mu \partial^\mu (\partial_\nu D^\nu) + \frac{\delta D_\mu (A_\mu^\omega)}{\delta \omega}|_{\omega=0} \partial^\mu (\partial_\nu A^\nu). \tag{3.20}$$

The gauge fixing and Faddeev-Popov terms then become in the modified scheme

$$\begin{aligned} \mathcal{L}_g &= -iBD_\mu \partial^\mu (\partial_\nu A^\nu) - \bar{c}[D_\mu \partial^\mu (\partial_\nu D^\nu) + \frac{\delta D_\mu (A_\mu^\omega)}{\delta \omega}|_{\omega=0} \partial^\mu (\partial_\nu A^\nu)]c \\ &= -iBD_\mu \partial^\mu (\partial_\nu A^\nu) - \bar{c} D_\mu \partial^\mu (\partial_\nu D^\nu) c. \end{aligned} \tag{3.21}$$

In this last step, we used the fact that the gauge fixing condition

$$D_\mu \partial^\mu (\partial_\nu A^\nu) = 0 \tag{3.22}$$

is equivalent to

$$\partial_\nu A^\nu = 0 \tag{3.23}$$

in Euclidean theory if the Gribov-type complications are absent and thus the inverse of the operator $D_\mu \partial^\mu$ is well-defined. We thus have the path integral formula in the modified scheme

$$\begin{aligned} &\int \mathcal{D}A_\mu^a \mathcal{D}B^a \mathcal{D}\bar{c}^a \mathcal{D}c^a \exp\{-S_{YM}(A_\mu^a) + \int dx[-iBD_\mu \partial^\mu (\partial^\nu A_\nu) - \bar{c} D_\mu \partial^\mu (\partial_\nu D^\nu) c\} \\ &= \int \mathcal{D}A_\mu^a \mathcal{D}B^a \mathcal{D}\bar{c}^a \mathcal{D}c^a \exp\{-S_{YM}(A_\mu^a) + \int dx[-iB(\partial^\mu A_\mu) + \bar{c}(-\partial_\mu (D^\mu c)]\} \end{aligned} \tag{3.24}$$

after the re-definition of auxiliary variables

$$BD_\mu \partial^\mu \rightarrow B, \quad \bar{c} D_\mu \partial^\mu \rightarrow \bar{c} \tag{3.25}$$

which leaves the path integral measure invariant. This last re-definition is allowed only when the operator $D_\mu \partial^\mu$ is well-defined, namely, in the absence of Gribov-type complications in Euclidean theory.

We have illustrated that the apparent "massive gauge field" in the classical level has no intrinsic physical meaning. It can be interpreted either as a classical massive (non-gauge) vector theory, or as a gauge-fixed effective Lagrangian for a massless gauge field. In the framework of path integral, we have a certain freedom in the choice of the path integral measure: One choice of the measure

$$\begin{aligned}
&\int d\mu \exp\{\int dx[-\frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu)^2 - \frac{m^2}{2}A^a_\mu A^{a\mu}]\} \\
&\equiv \int \mathcal{D}A_\mu \frac{1}{\int \mathcal{D}g \exp[-\int \frac{m^2}{2}(A^{ag}_\mu)^2 dx]\}} \\
&\times \exp\{\int dx[-\frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu)^2 - \frac{m^2}{2}A^a_\mu A^{a\mu}]\}
\end{aligned} \tag{3.26}$$

gives rise to a massless gauge theory, and the other choice

$$\begin{aligned}
&\int d\mu \exp\{\int dx[-\frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu)^2 - \frac{m^2}{2}A^a_\mu A^{a\mu}]\} \\
&\equiv \int \mathcal{D}A_\mu \exp\{\int dx[-\frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu)^2 - \frac{m^2}{2}A^a_\mu A^{a\mu}]\}
\end{aligned} \tag{3.27}$$

gives rise to a massive *non-gauge* vector theory. A somewhat analogous situation arises when one attempts to quantize the so-called anomalous gauge theory: A suitable choice of the measure with a Wess-Zumino term gives rise to a consistent quantum theory, if not renormalizable.

From a view point of classical-quantum correspondence, one can define a classical theory uniquely starting from quantum theory by considering the limit $\hbar \to 0$, but not the other way around in general.

In the context of the present interpretation of massive gauge fields, the massive gauge fields generated by the Higgs mechanism are exceptional and quite different. The Higgs mechanism for Abelian theory, for example, is defined by (in this part, we use the Minkowski metric with $g_{\mu\nu} = (1, -1, -1, -1)$)

$$\mathcal{L} = (D^\mu\phi)^\dagger D_\mu\phi - \mu^2|\phi|^2 - \lambda|\phi|^4 - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \tag{3.28}$$

which is manifestly gauge invariant with $D_\mu = \partial_\mu - igA_\mu$. The mass $m = gv$ for the gauge field is generated after the spontaneous symmetry breaking of gauge symmetry defined by

$$\phi(x) \equiv \phi'(x) + v/\sqrt{2} \tag{3.29}$$

with

$$v^2 = -\mu^2/\lambda \tag{3.30}$$

for $\mu^2 < 0$. In this procedure, all the terms in the Lagrangian including the mass term generated by the Higgs mechanism are gauge invariant. Consequently, our argument discussed so far does not apply to the present massive vector particle whose mass is generated by the Higgs mechanism. It is quite satisfactory that the Higgs mechanism has an intrinsic physical meaning even in our extended scheme of quantum gauge symmetry.

4 Dynamical generation of gauge fields

It is a long standing question if one can generate gauge fields from some *more* fundamental mechanism. In fact, there have been numerous attempts in the past to this effect. To our knowledge, however, there exists no definite convincing scheme so far. On the contrary, there is a no-go theorem or several arguments against such an attempt[7][8]. We here briefly comment on this issue from a view point of our extended scheme of quantum gauge symmetry.

Apart from technical details, the basic argument against the "dynamical" generation of gauge fields is that the Lorentz invariant positive definite theory cannot simply generate the negative metric states associated with the time components of massless gauge fields. In contrast, the massive "gauge fields" for which one can define a rest frame of the particle and thus avoid the appearance of negative metric, could be generated dynamically. In general, the dynamical generation of the Lagrangian of the structure

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu)^2 - f(A^a_\mu) \tag{4.1}$$

does not appear to be prohibited by general arguments so far. Here the term $f(A^a_\mu)$ is Lorentz invariant but not invariant under the local gauge symmetry and thus breaks the gauge symmetry explicitly; $f(A^a_\mu) = \frac{m^2}{2}(A^a_\mu)^2$ is the simplest example.

Here comes the issue of interpretation of the induced Lagrangian (4.1). If one regards (4.1) as a quantum theory from the beginning, what one generates is simply a non-gauge theory: This is also the case if one evaluates a general S-matrix and looks for the possible poles corresponding to massless gauge particles.

However, one might consider that the induced Lagrangian such as (4.1) is a *classical* object which should be quantized anew: In terms of path integral language, the Lagrangian is induced when one integrates over the "fundamental" degrees of freedom, and one need to perform further path integral over the induced Lagrangian anew. If one takes this latter view point, one could regard the part of $f(A^a_\mu)$, which breaks classical gauge symmetry, as a gauge fixing term in the modified quantization scheme above. In this latter interpretation, one might be allowed to say that massless gauge fields are generated dynamically. Although a dynamical generation of pure gauge fields is prohibited, a *gauge fixed* Lagrangian may be allowed to be generated. (In any case, if one should really like to produce gauge fields dynamically in spite of general arguments against it[7][8], there appears to be not much choice but presumably the one we are discussing.) The mass for the gauge field which has an intrinsic unambiguous physical meaning is then further induced by the spontaneous symmetry breaking of the gauge symmetry thus defined (the Higgs mechanism).

We note that the mechanism for generating gauge fields by the violent random fluctuation of gauge degrees of freedom at the beginning of the universe, which was advocated by Nielsen and his collaborators[9], is based on the renormalization group flow starting from an initial chaotic theory. In such a scheme, it is natural to think that one is always dealing with quantum theory, and thus no room for our way of re-interpretation of the induced theory. In their scheme, however, the question remains how the massive particle without negative norm, for example, evolves into the massless gauge field with indefinite metric in a manifestly Lorentz invariant manner. To be precise, an example of massive Abelian gauge field is analyzed in *compact* gauge theory in Ref.[9]. One thus still has a certain freedom of gauge fixing when one de-compactifies the theory, and consequently one might be able to apply our analysis to their scheme also.

5 Discussion

We have argued that the classical action by itself, when it comes to the issues of local gauge symmetry, does not specify uniquely what kind of theory one obtains when the theory is properly quantized. In this general interpretation of the classical action, the quantum gauge symmetry (BRST symmetry) can be defined for a much wider class of theories than pure classical gauge theory, such as Maxwell field and Yang-Mills fields. Classical gauge symmetry is sufficient to generate quantum gauge symmetry (up to quantum anomalies), but it is not necessary in general.

The BRST symmetry is thus quite universal. This universality presumably arises from the fact that the essence of BRST symmetry is quite simple; geometrically, it is defined as the translation and scale transformations of a superspace coordinate specified by the real element of the Grassmann algebra[10]

$$\begin{aligned} &Q:\theta\rightarrow\theta+\lambda, \quad (BRST \;\; charge) \\ &D:\theta\rightarrow e^{\alpha}\theta, \quad (ghost \;\; number \;\; charge) \end{aligned} \tag{5.1}$$

where θ and λ are the real elements of the Grassmann algebra and α is a real number. Algebraically

$$\begin{aligned} &[\lambda Q,\lambda Q]=0, \\ &[D,Q]=Q, \\ &[D,D]=0. \end{aligned} \tag{5.2}$$

In conclusion, we hope that the observation in the present note will stimulate further thinking on the real nature and possible origin of gauge fields, the most profound notion of modern field theory.

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